Two from the Putnam

The William Lowell Putnam Mathematics Competition is the premier North American college mathematics competition. More than half of the participants score zero points on the twelve problems they are given six hours to solve (in three-hour increments). Here are the two problems I could solve in the three years that I took this exam.

1998 A-1

A right circular cone has base of radius 1 and height 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the side-length of the cube?

1996 A-1

Find the least number A such that for any two squares of combined area 1, a rectangle of area A exists such that the two squares can be packed in the rectangle (without interior overlap). You may assume that the sides of the squares are parallel to the sides of the rectangle.

Just for fun, here is the hardest problem from the 1998 exam, based on the number of solvers. Just 12 of the top 199 contestants solved it!

1998 A-6. Let A, B, C denote distinct points with integer coordinates in $\mathbb{R}^2$. Prove that if

$$(|AB| + |BC|)^2 < 8 \cdot [ABC] + 1$$

then A, B, C are three vertices of a square. Here $|XY|$ is the length of segment XY and $[ABC]$ is the area of triangle ABC.