

l'Hôpital's rule

l'Hôpital's Rule is a useful tool for computing some limits, but care must be taken when applying it.

1. Use l'Hôpital's rule to find

$$\lim_{x \rightarrow 1} \frac{x^x - x}{1 - x + \ln x}.$$

(This is an exercise in a textbook written by Euler. Be sure to explain why we are allowed to use l'Hôpital's rule.)

2. Explain why l'Hôpital's rule *cannot* be used to find

$$\lim_{x \rightarrow \infty} \frac{x + \sin x}{x}.$$

Find the limit using a different technique.

3. Explain why l'Hôpital's rule *cannot* be used to find

$$\lim_{x \rightarrow \infty} \frac{x + \cos x \sin x}{e^{\sin x} (x + \cos x \sin x)}.$$

Find the limit using a different technique. (This is a surprisingly difficult problem. I encourage you to ask a professor and/or consult outside sources, but I wouldn't be surprised if no one could help you!)