Week 1. Proposed by Matthew McMullen.

Show that $\sqrt{2009} + \sqrt{2010}$ is a root of a fourth degree polynomial with integer coefficients. Is there a non-zero polynomial with integer coefficients and of degree less than four that $\sqrt{2009} + \sqrt{2010}$ is a root of?

Week 2. Proposed by Matthew McMullen.

(a) Let $m \geq n \geq 0$. Show that

$$\sum_{i=n}^{m} \binom{i}{n} = \binom{m+1}{n+1}.$$

(b) Find

$$\sum_{i=1}^{1729} \left( \frac{3739 - i}{2010} \right) \quad \text{and} \quad \sum_{i=1}^{1729} i \left( \frac{3739 - i}{2010} \right).$$

Week 3. Proposed by Matthew McMullen.

If $N$ is a positive integer with at least two prime divisors, define the delta value of $N$ to be $p - q$, where $q < p$ are the two largest prime divisors of $N$. Find the previous five years and the next five years with the same delta value as 2010.
**Week 4. Proposed by Matthew McMullen.**

Let \( F_1 = 1 = F_2 \) and \( F_n = F_{n-2} + F_{n-1} \) for \( n \geq 3 \). (So \( F_n \) is the \( n \)th Fibonacci number.) Find all \( n \) such that \( F_n = n^2 \).

**Week 5. Proposed by Matthew McMullen.**

(a) When I type \( i^i \) into my TI-83 calculator, it gives me 0.2078795764. When I type in \((-i)^i\), it gives me 4.810477381. What are the exact values of these numbers? More generally, how would you “make sense” of \( z^w \), where \( z \) and \( w \) are complex numbers (and \( z \neq 0 \))?

(b) Classify all complex numbers \( z \) and \( w \) with \( |z| = 1 \) and \( z^w \in \mathbb{R} \).

**Week 6. Proposed by Matthew McMullen.**

Suppose

\[
\int_0^a \frac{1}{\sqrt{1 + \sqrt{x}}} \, dx = 2010.
\]

Find, with minimal computational aid, the first two digits of \( a \).

**Week 7. 2009 Ohio MAA Student Team Competition.**

Let \( P \) be a point picked at random inside the equilateral triangle \( ABC \). What is the probability that the angle \( \angle APB \) is an acute angle?

**Week 8. Proposed by Matthew McMullen.**

Find

\[
\sum_{n=1}^{\infty} \frac{2n - 1}{(4n - 1)!}.
\]

Let $T$ be the region bounded by an isosceles triangle. Mathematically describe all ways of dividing $T$ into two equal-area pieces using a straight line.

Week 10. (a) Proposed by Ryan Berndt; (b) 1999 ECC Problem 5.

(a) Show that the formula
\[ \int_{-1}^{1} p(x) \, dx = p(-\sqrt{3}/3) + p(\sqrt{3}/3) \]
yields exact results for polynomials of degree three or less.

(b) (i) Find the points $x_1$ and $x_2$ so that the formula
\[ \int_{0}^{1} p(x) \, dx = p(x_1) + p'(x_2) \]
yields exact results for polynomials of degree two or less.

(ii) Determine the error in using the resulting formula for a third degree polynomial $p(x)$ with leading coefficient 1.