

Let's Be Fair About This: Strategic Thinking in Community

- Divide this cookie fairly between two people:



- Definitions:
 1. Fair:
 2. Envy-free:
- Divider-chooser algorithm (continuous, 2 players):
 1. Procedure:
 2. Fair?
 3. Envy-free?

- Lone divisor algorithm (continuous, 3 players):
 1. Procedure:
 - Player 1 proposes division
 - Players 2 & 3 mark approved pieces
 - If possible, sensibly distribute approved pieces
 - If Players 2 & 3 only approve same single piece
 - One of the other two unapproved pieces is given to Player 1
 - Players 2 & 3 regroup the remaining two pieces and do divider-chooser



2. Fair?
3. Envy-free?

- Last diminisher algorithm (continuous, n players):
 1. Procedure:
 - An ordering for the participants is decided
 - The first participant proposes a piece which they think is fair.
 - One at a time, the other players may choose to accept or reject it.
 - If they reject it, they must *diminish* that piece to a smaller *fair* piece.
 - Once all players have accepted/rejected, the piece is given to the *last diminisher*. They provide no further input.
 - This repeats until two remain. They do divider-chooser.
 2. Fair?
 3. Envy-free?

- Divide this property among four people:



Discrete Fair Division: Method of Sealed Bids (m indivisible items, n players)

	P1:	P2:	P3:
i. Bid for Item 1: PUPPY			
ii. Bid for Item 2: CAR			
iii. Bid for Item 3: CONDO			
iv. Bid for Item 4: N/A			
v. Total value (sum of rows i. through iv.)			
vi. Fair share (row v. divided by number of participants)			
vii. Items assigned (highest bid wins)			
viii. Total value of items assigned in row vii.			
ix. First settlement cash (row vi. minus row viii.)			
x. Second settlement cash (add across row ix., make positive, divide by number of participants)			
xi. Final settlement (add rows vii., ix., and x.)			
xii. Personal cash value (add rows viii., ix., and x.)			

	P1:	P2:	P3:
i. Bid for Item 1:			
ii. Bid for Item 2:			
iii. Bid for Item 3:			
iv. Bid for Item 4:			
v. Total value (sum of rows i. through iv.)			
vi. Fair share (row v. divided by number of participants)			
vii. Items assigned (highest bid wins)			
viii. Total value of items assigned in row vii.			
ix. First settlement cash (row vi. minus row viii.)			
x. Second settlement cash (add row ix., make positive, divide by number of participants)			
xi. Final settlement (add rows vii., ix., and x.)			
xii. Personal cash value (add rows viii., ix., and x.)			

Voting Theory

- Fairness Criteria:
 1. *Majority Criterion*: If a candidate receives a majority (more than half) of the 1st place votes, then that candidate should win the election.
 2. *Condorcet Criterion*: If a choice is preferred in every one-to-one comparison with the other choices, then that choice should win.
 3. *Independence of Irrelevant Alternatives Criterion*: Disqualifying a losing candidate should not change the winner
 4. *Monotonicity Criterion*: If voters change their votes to increase their preference for a choice, it should not harm that choice's chances of winning.

- Voting Systems:
 1. *Plurality*: Candidate with most 1st choice votes wins.
 2. *Pairwise comparisons*: Compare each pair of candidates. +1 for win, +1/2 for tie. Highest total wins.
 3. *Instant runoff*: Candidate with *majority* of 1st choice votes wins. If none exists, eliminate option with fewest 1st choice votes. Repeat until remaining candidate with majority of 1st choice votes identified. They win!
 4. *Borda Count*: Points for each place, e.g., 1 point for last choice, 2 points for 2nd-to-last choice, etc., awarded from each ballot. Candidate with highest total wins.

- Arrow's _____ Theorem:

Group 1 preference schedule:

	1	3	3	3
1 st choice	A	A	C	B
2 nd choice	C	B	B	A
3 rd choice	B	C	A	C

Election 1:	Winner	Fairness violation
Plurality		

Group 2 preference schedule:

	5	5	6	4
1 st choice	D	A	C	B
2 nd choice	A	C	B	D
3 rd choice	C	B	D	A
4 th choice	B	D	A	C

Group 2:	Winner	Fairness violation
Pairwise Comparisons		

Denny's closed:

	5	5	6	4
1 st choice	A	A	C	B
2 nd choice	C	C	B	A
3 rd choice	B	B	A	C

Group 3 preference schedule:

	37	22	12	29
1 st choice	A	B	B	C
2 nd choice	B	C	A	A
3 rd choice	C	A	C	B

Group 3:	Winner	Fairness violation
Instant runoff		

10 people change from B-A-C to A-B-C:

	47	22	2	29
1 st choice	A	B	B	C
2 nd choice	B	C	A	A
3 rd choice	C	A	C	B

Group 4 preference schedule:

	51	25	10	14
1 st choice (3 points)	B	A	D	C
2 nd choice (2 points)	A	D	A	A
3 rd choice (1 point)	C	C	C	D
4 th choice (0 points)	D	B	B	B

Group 4:	Winner	Fairness violation
Borda Count		