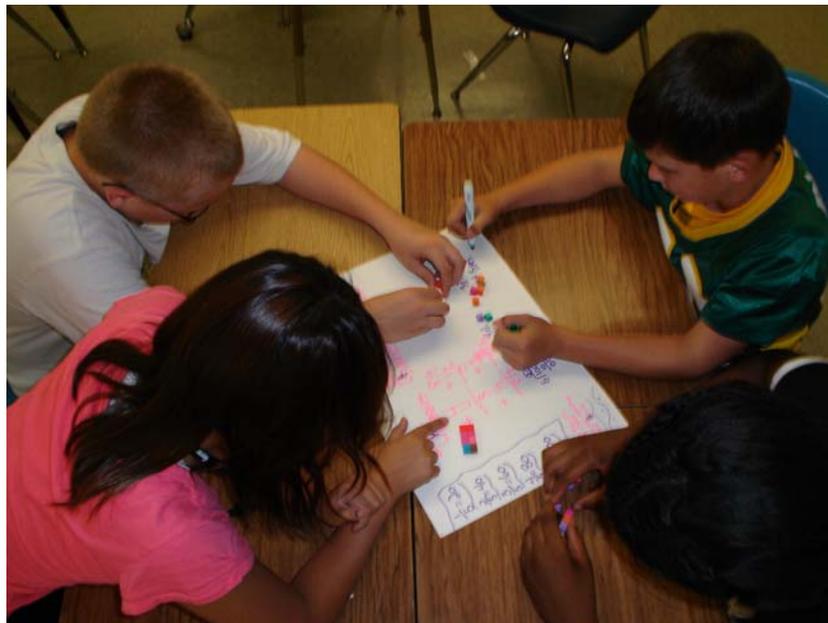


# CGI: A New Way to Teach Fractions to Middle Schoolers

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## Abstract

*Today's teachers are expected to challenge students so that they develop a deep understanding of math. Fractions are often hard for middle school students to conceptualize. The purpose of this study was to find a better way to teach multiplying and dividing fractions to our 6<sup>th</sup> grade students. We implemented Cognitively Guided Instruction (CGI) in a 15-day unit; this approach gives students freedom to invent strategies to solve complex problems. Through CGI, teachers analyze student understanding by looking at their strategies. Our research was conducted in two different classroom environments, one rural and the other suburban. We administered a pre and posttest, and students wrote in journals on a daily basis. From the results of our pre and posttests, CGI did not greatly affect our students' ability to solve fraction problems correctly, but growth was revealed through journal entries that demonstrated improved mathematical thinking.*

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# Introduction

*(Two students solving a math problem together)*

**Sandy:** I would put how many  $2\frac{1}{2}$ s go into 50; like keep adding it up.

**Ryan:** What about long division?

**Sandy:** I forget how to do that.

**Ryan:** How many  $2\frac{1}{2}$  groups can we split 50 into? We can try  $2\frac{1}{2} \times 27$  to see if it gets close to 50. That is 67.5 so it's too much.

**Sandy:** Try  $2\frac{1}{2}$  times 21 but that equals 50.1.

**Ryan:** Wait that does not make sense because 5 has to be in the tenths place because of 2.5 when you multiply. So  $2\frac{1}{2}$  times 21 equals 52.5

**Sandy:** So, the answer has to be smaller. What is  $2\frac{1}{2}$  times 20?

**Ryan:** Hey, it's 50. So that means that 50 divided by  $2\frac{1}{2}$  is 20.

This conversation is surely different than the ones that the authors, Trish and Scott, remember from their middle school days. We remember being taught to “multiply the top numbers and then the bottom numbers” when multiplying fractions and “do not ask why, just flip and multiply” when dividing fractions. What relationship do these algorithms have to the actual operations? As students, we were never asked to worry about such things. However, these algorithms do not promote understanding unless students know where they come from. We were determined to find a better way to teach fractions than we had been exposed to as students.

Since we are both middle school math teachers, we decided to team up to do an action research project in order to see if we could find a better way to teach fractions. Trish (subsequently referred to as “T”) teaches in a [suburban middle school](#) with a heterogeneous population (26% qualify for free or reduced priced lunch; 76.7% of the students are White, 15.7% Black, 4.6% Multiracial, and 1.6% Hispanic). In contrast, Scott teaches in a [rural middle school](#) where only 16.8% are eligible for free or reduced priced lunch and 97% of the students are White.

Fractions are an intimidating part of the 6<sup>th</sup> grade math curriculum for students. We surmised that fractions become difficult for students when they can no longer visualize them. We landed on Cognitively Guided Instruction (CGI) as a way to improve our students’ mathematical thinking. This method has its roots in early elementary math but in theory could be used with older students (Carpenter, Fennema, Franke, Levi, & Empson, 1999). In using CGI, students are given a series of real world problem solving situations and are expected to create strategies. Students present their strategies to the class, which provides an opportunity for learning from peers. In CGI, the teacher’s role is that of a facilitator helping the students model the problem and reason to find a

strategy. At no time is the teacher responsible for teaching the right way to answer the problem. The students learn on their own or from one another.

Scott and I not only have different student demographics, but we also have different approaches to teaching math. I use an exploratory style which is similar to CGI. In my district, *Connected Math* (2004) is our primary resource. This program encourages students to explore multiple strategies and presents students with real world connections in problem situations. Because of this, I decided to use all of my classes for the study. Scott, on the other hand, decided to use just one class for the study because he uses a traditional style of teaching, which is very different than CGI. He uses the text, *Mathematics: Applications and Connections* (1999), which focuses on specific routines and procedures students learn from the teacher. This teaching approach could not be more different from CGI. CGI focuses on the process of mathematical thinking that students experience in exploring mathematics (Bright, Bowman, & Vace, 1998).

## Theoretical Framework

It is not by chance that CGI includes the word, “cognitive.” Jean Piaget, a psychologist well known for changing the way we understand cognition, described four stages of cognitive development. During middle school, students are in the concrete-operational stage of thinking. They use “hands-on” activities to make sense of the physical world. In math, that would involve using manipulatives to have a better understanding of mathematical ideas. A study on fractions done with middle school students found that the use of manipulatives significantly improved students’ basic understanding of fractions (Reys, Kim, & Bay, 1999).

While Piaget focused on the individual’s active construction of meaning, other cognitive psychologists drew attention to metacognition, thinking about one’s own thinking. Metacognition is used to monitor and regulate cognitive processes like learning and problem solving. Individuals plan a way to approach a learning task, monitor comprehension, and evaluate their thinking and learning processes (Woolfolk, 1998).

Metacognitive knowledge is important in CGI. Students should plan how to start answering a word problem, figure out what strategies to use, and what types of resources they need. The next skill is monitoring. Students make sure things are making sense and that they are using their time efficiently. The last essential skill in metacognitive knowledge is evaluation. Students are examining the strategies they used to solve the problem and making sure the problems have been solved adequately.

Carpenter, Fennema, Franke, Levi, and Empson (2000) developed CGI based on their work involving children’s and teachers’ cognition about mathematical thinking and teachers’ instructional decision-making. The researchers found that teachers knew a great deal on an intuitive level about student thinking but their intuitions were too fragmented to use successfully in making instructional decisions. Thus, they decided that a framework for teachers to use to focus effectively on student understanding was needed (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989).

Communication is an essential part of CGI. Lev Vygotsky's sociocultural theory provides an important link between communication and learning. His theory supports peer interaction in learning, which is vital to the CGI format (as cited in Woolfolk, 1998). Students talk, draw, and write about their methods in solving a problem (Fennema, Carpenter, & Franke, 1992). This encourages them to reflect and think about their strategies. After students discuss their strategies with their peers, they then present their methods to the class. Then the teacher questions the students to make the strategy clearer to the class. Next, students are asked to focus on similarities in comparing strategies that other students used. Any strategy that presents a correct answer and can be explained is thought to be correct and important. When students present their work to the class, the teacher can also informally assess students' understanding and use it to make instructional decisions about what problems the students are ready for next.

There is substantial evidence of the effectiveness of CGI for elementary school populations but not too much for middle school (Carpenter, Ansell, Levi, Franke, Fennema, & Weisbeck, 1993, Villesenor & Kepner, 1993, Carpenter & Levi, 2000). While CGI advocates believe it can be used through sixth grade, our survey of literature revealed a lack of research in the upper grades.

## Procedure

We took the structure of CGI and adapted it to our middle school classrooms. After a five-minute warm-up, students were given a CGI problem. While devising our lesson plan, we utilized CGI's categories to create problems. (Carpenter, Fennema, Franke, Levi, & Empson, 1999) While the categories were developed for elementary multiplication and division of whole numbers, we used them for fractions. *Measurement Division* is a category in which the students know the total and how much belongs in each group but the question refers to finding the total number of groups.

**Example: Tony has  $\frac{7}{8}$  of a pie. He wants to give each friend a  $\frac{1}{4}$ . How many friends can he feed?**

The other category, *Partitive Division*, the total and number of groups is known, but the student tries to find how much or how many are in each group.

**Example: I have  $1\frac{3}{4}$  yard of ribbon that makes 9 bows. How much ribbon does it take to make one bow?**

Heterogeneous groups of 4-5 students devised strategies to solve CGI problems. In all, there were 15 lessons. On odd days the problems fell in the *Measurement Division* category and on even days in the *Partitive Division* category. Students illustrated these strategies on posters through pictures and numeric expressions. Groups worked for 10-15 minutes and were expected to present their strategies. We chose one representative per group to explain the method that the group used. We then questioned the presenters to get a clearer picture of their group's thought process. During these 15-minute presentations, all students were expected to learn from one another and to see the problem from different perspectives.

At the end of each class, students were given ten minutes to answer a second CGI problem in a journal. This journal was a packet of graph paper where students could work out problems and use the boxes to help them illustrate their reasoning. Students were also asked to justify their reasoning by writing below their work on the graph paper. In their journal, students included illustrations, numerical expressions, and an explanation of their strategies used to solve the problem. To meet the needs of all students, we provided differentiation in the journal question. The journal problem was presented with numbers at a grade-appropriate level. Students who felt like they were struggling with the first CGI problem were able to choose an easier set of numbers to work with. Students who felt confident with the first CGI problem had the option to challenge themselves in their journal by working with a harder set of numbers. Since we wanted to focus on giving students the opportunity to try out their own strategies, we did not give direct feedback on whether their solutions were correct. Our purpose in having students work out journal entries was to track their thought processes as they learned.

Date: \_\_\_\_\_  
 Problem: \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_

**Today's Thinking:** Explain your thinking about the mathematical problem by drawing pictures and describing what you are thinking clearly. Be sure to include detailed explanations and pictures that clearly show your thinking.

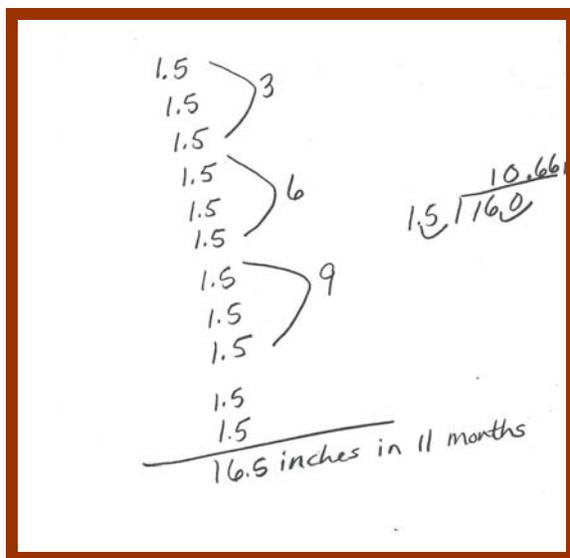
[A large grid of graph paper for working out the problem.]

***...journal was a packet of graph paper where students could work out problems and use the boxes to help them illustrate their reasoning.***

## Our Results

To begin our study, we gave an 11 item pretest. Scott and I graded them for *reasonableness*, which we defined as using a strategy that accurately represents the problem or situation through pictures and/or calculations. Students received credit if they used a sensible strategy whether or not they arrived at the right answer (see problem below).

**PROBLEM:** *Happy Valley needs 16 inches of rain annually to avoid drought conditions. If the average monthly rainfall is  $1\frac{1}{2}$  inches, how many months will it take to exceed the 16 inches needed?*



**Some students found that it would take exactly  $10\frac{2}{3}$  months to reach 16 inches; therefore, anything more than that would exceed the drought conditions. This would be considered a correct answer. However, some students calculated that 11 months would completely exceed this drought condition, which is also true.**

### My Results

My students were challenged by the pretest questions. My average pretest score was 2 out of a possible 11 questions (range 0-7). At the outset, students in my classes were somewhat uncomfortable with CGI. It took a few days for students to get used to representing solutions with pictures and manipulatives. Students also expressed some nervousness in presenting their strategies to the class. After three days, most students transitioned successfully into the CGI program. For the most part, students were engaged in group discussions when creating their posters displaying strategies that the group used. As the unit progressed, most students did not need blocks to help solve the problems and instead utilized strategies that they created throughout the unit.

In my three classes, as time passed, the groups became more effective in analyzing problems and creating strategies. Students learned much from working with their groups because the conversations those students had in the groups challenged them. No one in the group wanted to feel uncomfortable when presenting, which helped engage most

students in making sure they understood. Presentations of strategies were certainly effective for the presenters because it forced them to clarify their thoughts about strategies. However, I am not sure that students benefited from watching other presentations. Students did not immediately apply strategies from the presentations to their own problem solving.

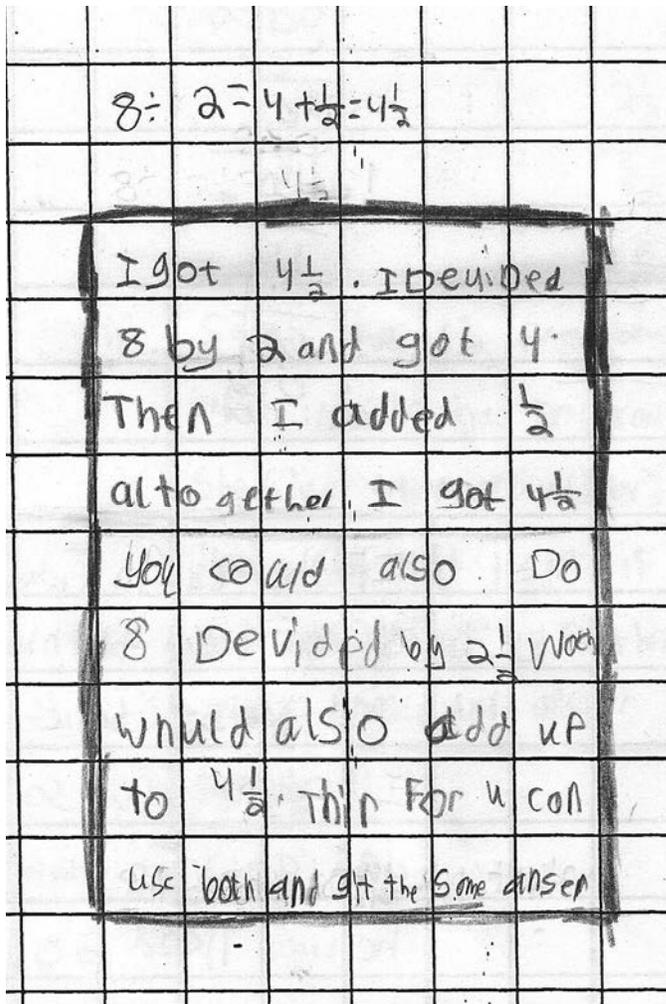


Throughout the unit, the groups in my classes tried many strategies. A common, successful strategy that worked for the students included drawing a model of the problem and demonstrating the solution. This was a less sophisticated strategy that students used to get an answer. Another common strategy included changing the fractions in the problems into decimals and using long division or multiplication. I even had students who estimated the fractions in the problem to come up with a reasonable approximation for the answer. These strategies were somewhat more sophisticated because they involved calculations and not direct modeling.

In analyzing my posttest data, the range of scores was 0-10. The mean of the scores also rose in all of the classes. The mean rose to about 5 questions correct out of 11. Some students made large increases in their scores from the pretest to the posttest. Thirty-four percent of my students raised their posttest score four points or more.

While student test scores did not demonstrate dramatic growth and mastery of multiplying and dividing fractions, the journal entries did give insight into the thought process of some the students. By observing students' strategies, I could identify how

close they were to understanding. I could also pick out any misconceptions students had at this point in their learning. For example, one student, Shawn, began the unit with a 0 on his pretest. He had struggled with math in the past and scored a Limited rating on his Ohio Achievement Test. Shawn's journals began with basic calculations involving, for example, addition, which were not always presented correctly. Through the 15 days of our CGI unit, Shawn struggled but showed signs of growth. For example, in one of his first journal entries, Shawn tried to add fractions together in order to divide which was unsuccessful.



**PROBLEM:** *Eight students had to share  $2\frac{1}{2}$  containers of glitter for a project. How much of a container will each student get to use?*

**Shawn's Strategy:** Instead of dividing  $2\frac{1}{2}$  by 8, Shawn switched the numbers and then divided. He showed 8 divided by 2 and got 4 and then tacked on the extra  $\frac{1}{2}$  that was left. At this point, he clearly does not show an understanding of fractions because not only did he invert the numbers that were divided, he didn't divide the remainder among the groups.

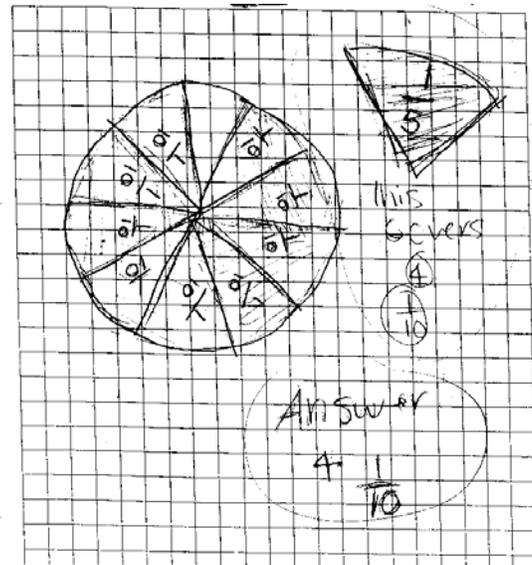
Further along in our unit, Shawn learned more about what it meant to multiply and divide fractions. He focused on visualizing what was happening in the problem which greatly helped him. In this problem, Shawn not only drew a picture to represent the problem, he also divided the remainder among the rest of the groups. This shows that he has an understanding of how to divide fractional parts.



In Scott's classroom, students were able to use [fraction circles](#). By using the circle, students could overlap parts of the circle to see how many parts could actually make a certain fraction. The students could really see the fractions and manipulate them as they figured out the problem. Another strategy that his students discovered was the adding of fractions. Many students added fractions to see how many parts could make a specific fraction. This repeated addition of fractions helped students model the multiplication problems. When some students used this strategy, it was clear that they understood the meaning of multiplying in the situations. Scott concluded that the addition led to more success on *Partitive Division* problems than it did on *Measurement Division* problems.

In Scott's class, posttest scores ranged from 0-8. The mean from the pretest scores to the post-scores went up less than a point. However, there was one student in particular where Scott saw a lot of growth. Nicole was a great math student--straight A's with a very keen mind for problem solving. Scott knew that this girl should do well with CGI. The student scored a one on the pretest, and this discouraged Scott. Nicole was totally against CGI. To her, it was not what she was used to. She was accustomed to the teacher presenting the topic on the board and then doing homework for practice. CGI was out of her comfort zone. During the group work, she was off-task and leaned on others to figure out strategies. Her first few journal entries did not include much explanation, just a couple of unclear pictures. The strategies were not clear.

As the unit continued, Scott was surprised by the growth that Nicole made. She was now enjoying CGI. She was becoming a leader and an advocate for CGI. Instead of leaning on others, she now had students leaning on her. Her confidence level increased tremendously. She got eight out of 11 questions correct on her posttest. Below are two of her journal entries, one from the beginning of CGI and the second, from the end of the unit.



***By using fraction circles, this student modeled the division of  $9/10$  by  $1/5$ .***

Date: 3-19-07

Problem: Max has  $\frac{7}{8}$  of a King Size Butter Finger Candy Bar. He shares it equally with his 3 friends. How much of the candy bar will each friend get?

Today's Thinking: Explain your thinking about the mathematical problem by drawing pictures and describing what you are thinking clearly. Be sure to include detailed explanations and pictures that clearly show your thinking.

Green

$\frac{7.5}{10}$

they each get:  
 $\frac{7}{8} \div 3 = \frac{7}{24}$

6: 8, 16, 24  
 3: 3, 6, 9, 12, 15, 18, 21, 24

**PROBLEM:** Max has  $\frac{7}{8}$  of a King Size Butterfinger candy bar. He shares it equally with his three friends. How much of the candy bar will each friend get?

**Nicole's Strategy:** Nicole got her answer by multiplying 7 by 3 and 8 by 3 to get  $\frac{21}{24}$ . She did not divide the  $\frac{7}{8}$  candy bar. Instead she developed an equivalent fraction for  $\frac{7}{8}$ . Nicole does not understand the concept of dividing up the fractional part.

Problem: You have 58 dollars. Each pack of gum costs  $\frac{1}{5}$  of a dollar. How many packs of gum can you buy?

Today's Thinking: Explain your thinking about the mathematical problem by drawing pictures and describing what you are thinking clearly. Be sure to include detailed explanations and pictures that clearly show your thinking.

10 Nice Job!

$5 = \frac{10}{50} = \frac{20}{100}$

$5 \times 20 = 1.00$

$\begin{array}{r} 4 \\ \times 58 \\ \hline 290 \end{array}$

290 packs of gum

**PROBLEM:** You have 58 dollars. Each pack of gum costs  $\frac{1}{5}$  of a dollar. How many packs of gum can you buy?

**Nicole's Strategy:** This time Nicole made  $\frac{1}{5}$  into  $\frac{20}{100}$ . From there, she figured out that  $\frac{1}{5}$  is the same as 20 cents. She then realized that 5 times 20 cents equals 1 dollar. She multiplied 5 by 58 dollars and came up with 290 packs of gum. This strategy showed her growth in fractions by being able to use ratios and multiplication to solve the problem correctly.

# Conclusions

After looking at our data, Scott and I were disappointed. We wanted test scores that showed large gains in understanding. We asked ourselves why students did not show the growth that we had anticipated. We identified some possible factors that may have affected our students' success:

- Length of time - the length of time in which students used CGI was much shorter than in any of the studies we read about. In fact, most of the studies we researched showed results of CGI after a year or more of use (Villasenor & Kepner, 1993, Fennema, Carpenter, & Franke, 1992). We only used CGI for 15 days.
- Structure change - If our students had been familiar with the method from elementary school, the transition would have been smoother and the students would have showed more growth. The students in Scott's class had very little time to make a huge transition in the way they were being taught. The 15 days of this unit were vastly different than the rest of the year, which may have affected the students' potential to gain mastery of these operations with fractions.

While our pretest and posttest results were not what we had hoped for, almost all students made some growth in their understanding. In fact, of my 56 students, only four did not show any growth. Students showed gains in their mathematical communication with each other as well. The group's conversations often displayed critical thinking about the mathematical concepts behind the strategies. I even had a class who got into a debate during the presentations about strategies that were successful.

Scott had fewer students show growth in their learning. He had seven students who did not show any improvement. This may have been because of the newness of the exploratory technique in teaching that Scott used in the CGI unit. Students were not used to creating multiple strategies to solve problems. Students were also not used to working without a calculator.

Scott and I agree that this unit did help students learn, but no teaching technique is an easy fix. No program is a magic solution to get all kids to understand. CGI presents one method that can be used to allow kids to use their critical thinking abilities to create strategies. It is clear that 15 days of CGI did not turn on a light bulb for every student. However, the focus on students' mathematical thinking is important. One thing that we took away from this experience is that while students are on the path to mastery, a teacher can learn a lot from students by analyzing their strategies. Through CGI, students' mathematical thinking became clearer which helped us meet the needs of our students.

## Next Steps

I appreciated the experience in using CGI and want to try to use it again. I want to try setting up my classroom routine similar to that of CGI. If I started the year with the CGI structure, students would get more out of the presentations and they would be more successful creating strategies and justifying them in front of the class. In this way, I can use some problems in my *Connected Math* (2004) series to prompt thinking from groups and to have groups present their thoughts. I am hoping that this metacognitive focus will help students learn more.

Scott did not like CGI. He was frustrated with his students being off task because they were not used working in groups. He thinks he may integrate more cooperative learning in his class in the future and maybe give CGI another try.

The world of education is certainly different than when Scott and I went to school. New challenges and expectations give our students an opportunity to think deeper than when we were students. The strategies our students created in those 15 days of CGI were more inventive and made more sense than the “don’t ask why, just flip and multiply” method that our teachers taught us.

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