"I Forgot that Quotient Meant to Divide so I Added Instead and Got the Wrong Answer": The Link between Math Vocabulary and Problem-solving

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Mathematics, vocabulary, problem-solving, middle school

Abstract
The purpose of this action research study was to find out if there is a correlation between mathematical vocabulary and problem-solving as well as to find vocabulary strategies that would effectively increase mathematical comprehension and conceptual understanding of middle school mathematics students. Four main strategies were implemented in the researcher's seventh grade classroom: personal glossaries, graphic organizers, word wall posters, and simile/fortune note cards. Students responded positively to vocabulary activities, and test scores indicated improved understanding of concepts.

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Introduction

"I thought the word quotient meant any problem, when it actually means a division problem. So a negative divided by a negative would be a positive."

"I didn't know the definition of congruent so I couldn't come up with an answer."

"Forgot what congruent meant so I guessed."

"I thought the word sum meant multiply so I did -3 x 4 which always comes out negative."

Not knowing vocabulary terms on the *Ohio Mathematics Achievement practice* test proved to be a problem for many students in my 7th grade math class. The quotations above are typical responses when my students are asked to explain their mistakes. Additionally, in class discussions over the past several years, I have noticed that students will struggle to answer questions when they do not understand the meaning of a mathematical word I use, such as *horizontal* or *slope*, or when they confuse similar terms, such as *intersect* and *y-intercept*. Based on my informal observations, I have suspected that word confusion leads to mathematical misunderstanding.

Literature Review

Mathematical reading is dense, and without understanding of specific vocabulary, many students struggle to understand concepts. Because of the high incidence of unfamiliar vocabulary in mathematics, teaching unknown words becomes central to mathematical literacy (Lee, 2007, p.125).

> Middle schoolers—students whose ability to reason abstractly is growing dramatically—in particular need to communicate mathematically in an increasingly sophisticated way. Deliberate and careful attention to acquiring and using the vocabulary of mathematics, with its wondrously specific technical language, is a must. This is a dramatic change, and one in which teachers play a crucial role" (Murray, 2004, p. 1).

Math teachers play such a crucial role because there is often a deficit in mathematics vocabulary in comparison to other content areas. Children are not often exposed to mathematical language in their homes and social environments (Murray, 2004). Additionally, Murray points out that recent math curriculum and standardized tests, "grounded in current research on teaching and learning, require mathematics writing competency, which in turn calls for an ever expanding vocabulary" (p. 11-12, 86). While understanding vocabulary is imperative to mathematical writing, the main goal of developing good vocabulary is to aid in comprehension, which, in math, helps students work through problems (Carter and Dean, 2006). Because mathematical reading is dense and each vocabulary word is conceptually-packed, it is imperative that teachers address vocabulary in their math classrooms in order to improve student comprehension and achievement.

Research has highlighted many effective vocabulary strategies, and it is important that teachers are familiar with a variety of them to meet the diverse needs of students in today's classrooms (Harmon, 2005, Carter and Dean, 2006, Lee, 2007). When teachers of mathematics step up and acknowledge their additional role as teachers of reading, students will be better able to understand and apply mathematics, hopefully increasing their test scores as well. Recognizing my roles as both a mathematics and reading teacher, I investigated the following questions:

> -Is there a strong relationship between my above-average students' understandings of specific vocabulary words and their performance on math problems?
> -If I provide vocabulary instruction (personal glossaries, word wall posters, graphic organizers, and similes) in math, do students come up with better definitions and give appropriate examples?
> -How do students respond to explicit vocabulary instruction in a mathematics class?

Methods

Participants

This research was conducted using three intact 7th grade mathematics classes totalling 55 accelerated math students at Middle School East. At Middle School East, there are four levels of mathematics: 7th grade math, accelerated 7th grade math, 7th grade honors math, and 8th grade honors math for 7th graders. Students are placed in the appropriate class based on their scores on the *IOWA Test of Basic Skills*' scores (2001). Students in the 7th grade accelerated math program, who were part of this study, scored in the 85th percentile or better on the quantitative section of the IOWA. While they are not considered gifted, they are above-average achievers in mathematics. Out of the 55 students, 28 are female and 27 are male. All students are Caucasian except for one African-American, three Asian, and two classified as "other."

Explicit Vocabulary Instruction

Personal Glossary

In order to provide numerous opportunities for my students to develop relationships between conceptual understanding and vocabulary knowledge, I implemented a variety of vocabulary teaching strategies in my mathematics classroom during the 2008-2009 school year. At the beginning of the year, I had students begin a notebook for in-class work. I directed them to set up a personal glossary at the back of these notebooks, counting off ten pages and attaching a "post-it" to mark the start of their glossaries (Murray, 2004). After the first unit pretest, I guided them through the process of adding vocabulary to their glossaries. I provided them six to ten words that we would be working with over the next few weeks, along with their definitions and corresponding examples. I also asked for student input when creating definitions, examples, or pictures that would help to address misconceptions that I had found on the pretests. At specific times, I required that students write certain vocabulary words and their definitions in their glossaries, while at other times throughout the year, students would choose vocabulary words with definitions and/or examples to add to their glossaries (see Figure 4).

Word Wall

In addition to math notebooks, I introduced word wall posters (Murray, 2004). These posters would be hung on the side wall of the classroom. Their purpose was to help individuals learn and understand new mathematics vocabulary and become more capable of answering questions and approaching problems as well as to give students a year-long reference to mathematical vocabulary. For each unit, students encountered specific mathematics vocabulary, so new words were constantly added to the word wall. Students, taking turns, were to complete an 8.5" by 11" poster for one word. The poster included the vocabulary word in large print, a definition using mathematical terms, a picture that demonstrated the meaning of the word, and an example. Students submitted a draft copy to me; together we made changes, and then the student gave me a final copy of their poster, which I laminated and hung on the classroom wall. Students were usually given three to five nights to complete a word wall poster (see Figure 5).

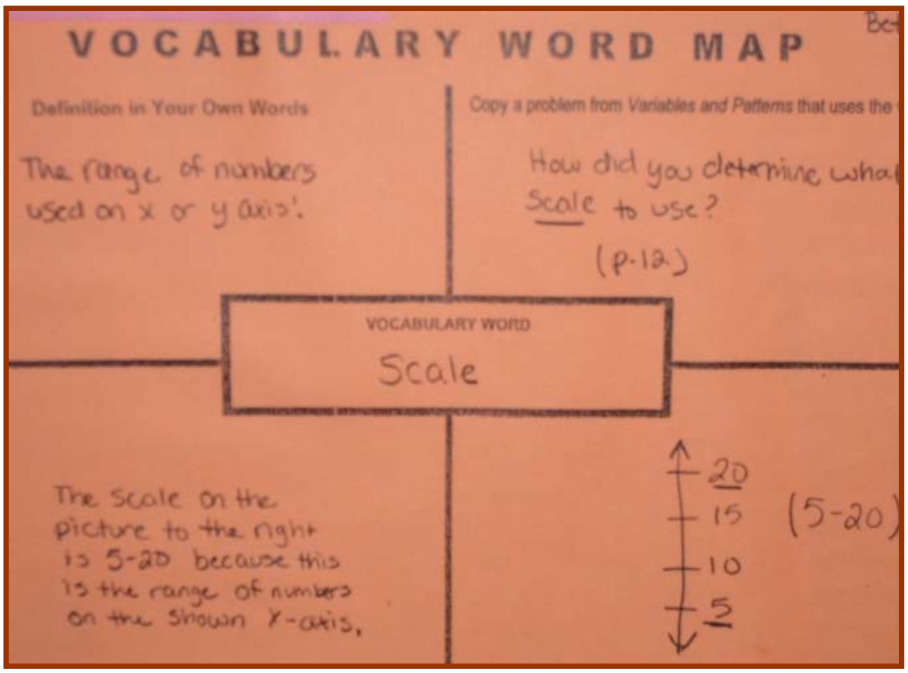


Figure 1 Graphic Organizer

Graphic Organizer

During the *Variables and Patterns* unit, I had students complete a graphic organizer (Murray, 2004, Harmon, 2005). Out of a list of ten to twelve vocabulary words from the unit, students chose five words that they did not know or struggled with on the pretest and wrote each word in the middle of a piece of paper that was divided into four sections. They had to: 1) write the definition, 2) give an example, 3) draw a picture, and 4) copy a problem from their textbook that asked a question or included that specific vocabulary word (see Figure 1). Students could get definitions from math dictionaries, websites, or their textbook.

Simile/Fortune Note cards

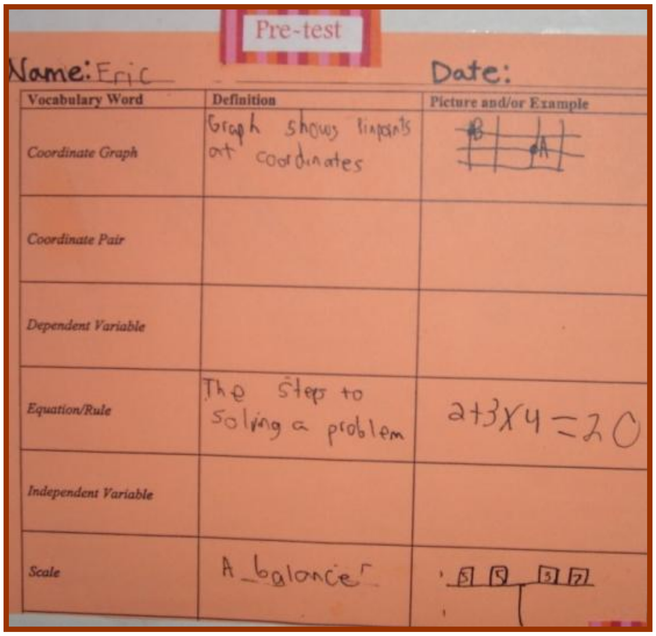
During the *Accentuate the Negative* unit, students learned to use figurative language as a way to learn mathematical terms (Rubenstein, 1996, Murray, 2004). Students needed to complete a note card for each word on the pretest that they missed. A word missed included an incorrectly matched word and definition, a word that they could not/did not provide an example or picture for, and a word whose picture/example column I marked because of an inaccurate example or picture. Students had to write the word and draw a picture or give an example on the front of the note card. They had three choices for the back of the card: They could create a simile, write a "fortune" for the word, or do a combination of the two (see Figures 8 and 9).

Throughout the year, I emphasized the meaning of vocabulary words, along with the corresponding concepts in whole-class discussion as well as in group, partner, and individual activities. Any time students asked questions about the meaning of a word, I asked the class to access their prior knowledge in order to answer these questions (Adams, 2003, Lee, 2007).

Data Gathering

I was interested in finding out if my students' vocabulary understanding, like that of their counterparts described in the professional literature (Carter and Dean, 2006, Capraro, 2006), correlated with their mathematical problem-solving performance. On unit post-tests, students' responses were scored on three different scales—one for defining a word, a second for giving an example, and a third for applying the math term correctly to problems. Students who got a definition, example, or application correct received a 3, partially correct a 2, and incorrect a 1. Therefore, the higher the student's average in each category, the better the student did at the specific task. Then, two correlations were computed: 1) a correlation between definition and application scores; and 2) a correlation between example and application scores.

In addition to determining correlations, I wanted to see if the vocabulary strategies I was using were effective. To determine the effectiveness of vocabulary strategies that can be used in teaching mathematics while supporting the development of math concepts, I ran paired t-tests for dependent groups. I wanted to determine whether or not an observed increase between students' pre- and posttest scores reached statistical significance. I ran these tests for definitions and for pictures/examples on two unit tests.



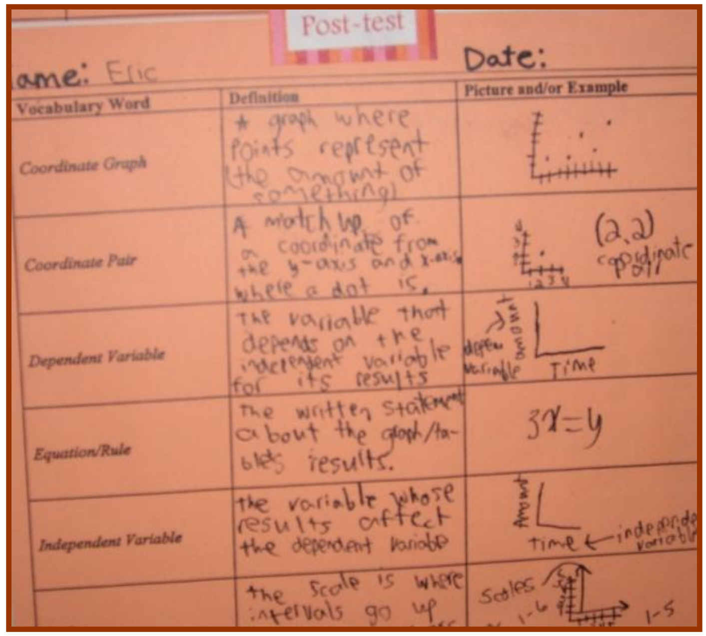


Figure 2 Unit 1 pre-test

Figure 3 Unit 1 post-test

Throughout the study, I also kept a log in which I recorded spontaneous student comments. I noted times when they used vocabulary correctly or when they became enthused about their new knowledge I augmented log notes with student surveys and interviews and kept track of excitement as well as the skepticism generated from vocabulary activities.

Findings

Question One: Is there a strong relationship between my above-average students' understandings of specific vocabulary words and their performance on math problems?

Correlations showed that a significant relationship existed between the definition and application scales, $r(55) = .862$, $p < .01$, indicating that if the student could correctly define the mathematical term, the student was more likely to apply the mathematical term. There was also a significant relationship between the definition scale and the example scale, $r(55) = .917$, $p < .01$, indicating that if the student could give a correct example of a mathematical term, the student was more likely to apply the mathematical term correctly in problem-solving. There was a stronger relationship between the example and application scales than definition and application scales; however, both were highly correlated and statistically significant ($p < .01$).

The results did not surprise me. When students do not know vocabulary definitions, they struggle to apply the terms to actual mathematical problems. There was also a correlation between being able to give an example of the mathematical term and being able to apply it to problems. The higher students scored on correct examples, the better they did on the actual application of the mathematics. This appears to be the case because if students really understand the mathematical definition of a term and/or can give an example of it, then it is likely that the student will know what to do when they encounter the term in a problem. My results are consistent with the theory and expert opinion documented in the professional literature.   Retention of tool vocabulary is critical for concept learning (Blachowicz and Fisher, 2000, as cited in Harmon, 2005).

Question Two: If I provide vocabulary instruction (personal glossaries, word wall posters, graphic organizers, and similes) in math, do students come up with better definitions and give appropriate examples?

Both vocabulary pre- and posttests yielded statistically significant differences; however, there were substantially bigger gains during unit one than unit two (see Table 1).

Unit	*Pretest average*	*Posttest average*	*Difference Scores*	*t*
Unit1:Variables and Patterns Definitions Range (0-10)	x = 1.24 SD = 1.4	x = 8.54 SD = 1.5	x = 7.3 SD =1.7	29.2*
Unit 1: Variables and Patterns Examples Range (0-10)	x =2.13 SD = 1.4	x = 9.15 SD = 1.0	x = 7.02 SD = 1.5	31.5*
Unit 2: Accentuate the Negative Definitions Range (0-7)	x = 5.15 SD = 1.8	x = 6.5 SD = 1.1	x = 1.35 SD = 1.8	5.2*
Unit 2:Accentuate the Negative Examples Range (0-7)	x = 3.29 SD = 2.1	x = 6 SD = 1.3	x = 2.71 SD = 2.4	7.7*

*** p < .05.**

Table 1 Pretest and Posttest Vocabulary Scores

Definitions of content-specific words and examples of those words and associated concepts were assessed on pre-and posttests, and there were significant improvements. Items on pre- and posttests were identical; therefore, practice effects may have factored into their improved scores. On the *Variables and Patterns* tests, the average number of correct answers, for both definitions and examples, increased by seven. By using graphic organizers as an intervention vocabulary strategy for *Variables and Patterns*, students seemed to have a better grasp on the meaning of the words at the end of the unit.

These findings are consistent with the literature in that using vocabulary strategies during math instruction will help promote conceptual knowledge and an understanding of math text (Lee, 2007). Murray (2004) found that concept maps allow students to organize information and vocabulary visually as well as make connections between concepts. In a study of the effects of two instructional models on the mathematical vocabulary of fourth grade students, Monroe (1997) found that the use of graphic organizers, along with in-depth discussion, effectively supported mathematical vocabulary development (as cited in Harmon, 2005). On the *Accentuate the Negative* test, the average number of correct answers increased by one for definitions and two for examples. The figurative language vocabulary strategy for *Accentuate the Negative* (student-made similes) also seemed to deepen student understanding. The smaller gain may be explained by the format of the test itself. There seemed to be less room for improvement on this test because students were able to match words with definitions, unlike the first test where they had to come up with their own definitions.

Of the different vocabulary activities I implemented (simile/metaphor note cards, graphic organizer, personal glossary, word wall, and “other”), 35% of students cited the simile/metaphor note cards as the vocabulary activity that helped them best understand the words.

Both the graphic organizer and simile/note card strategies seemed to be effective in improving test scores and helping students understand the mathematics better, although exposure to the words in the textbook and through class discussion as well as their deepened conceptual understanding throughout the unit may have also contributed to the stronger definitions, examples, and correct application of the vocabulary on the posttests. However, as cited in numerous studies, there tends to be a relationship between strengthening conceptual understanding and increased understanding of vocabulary (Johnson, 1944; Capps & Gage, 1987; Baker, Simons, & Kameenui, 1995, all as cited in Murray, 2004).

There certainly are limitations to this study: it is not possible to "prove" that mathematics achievement is a result of vocabulary instruction. What is clear, however, is that there is a strong correlation between vocabulary knowledge and mathematical applications.

Question Three: How do students respond to explicit vocabulary instruction in a mathematics class?

In a survey and reflection at the end of the semester, students wrote which vocabulary activity best helped them learn mathematical concepts. The most preferred activities were the word wall and the simile/fortune note cards. Thirty-five percent of students chose the word wall and 35% of students chose the simile/fortune note cards. Eleven percent of students chose the graphic organizer, 7% chose the personal glossary, 6% chose another additional activity or discussion, and 6% noted that 'nothing' best helped them learn mathematics through vocabulary work. Students seemed to like more creativity in mathematics vocabulary rather than copying down definitions. However, different students preferred different vocabulary activities as none stood out too clearly above the others; this suggests that using a variety of strategies is important because each reaches different types of learners.

On various surveys and interviews, students made comments citing the importance of vocabulary and explaining why they did or did not like certain vocabulary activities (See Table 2).

Table 2 Student Comments/Reactions to Vocabulary in Mathematics

Vocabulary Activity	Positive (+) Comments	Negative (-) Comments
Word Wall	***"[It] helped me learn a word in great detail." "The pictures and examples helped me a lot." "Typing it out and finding the definition helped me better know the word." "I always look up and have reference." "I like the word wall because I actually have to do it – look it up and find the definition and then tell other people what it is."***	
Simile/ Fortune Note Cards	***"It was fun to do and it made me think differently and harder." "It helps me understand the concept of what the words are saying." "It got us thinking about the actual definition, and in the other one [graphic organizer], we just had to copy down some stuff." "I really had to know them [the definitions] to write the fortunes."***	***"The fortunes were confusing at first but it helped."***
Graphic Organizer	***"The graphic organizer [was the best] because I had to give examples." "We thoroughly explained it [the term] and depicted the word."***	***"It was annoying. I saw no point to it."***
Personal Glossary	***"I can always look back."***	
Other	***"I like that you give us pre-tests. It helps me know what we're going to learn." "[It helps me best] when I actually use it [vocabulary.] Like absolute value, we had to learn the sign and then know what it meant."***	
Vocabulary in Math	***"You should [know math vocabulary] so you can explain situations using math terms and you have to understand them for work and school." "If we didn't know math vocabulary we wouldn't be able to answer math problems correctly."***	***"It [math vocabulary] helped me on the test but I don't know if it will help me in the real world." "Sometimes you need to know them [math vocabulary] in the real world and sometimes you don't."***

In looking back at various activities and discussions with my students as well as personal reflections, I, as the teacher, also found that there both positive and negative aspects of each vocabulary strategy. What follows are my own observations of strategy effectiveness:

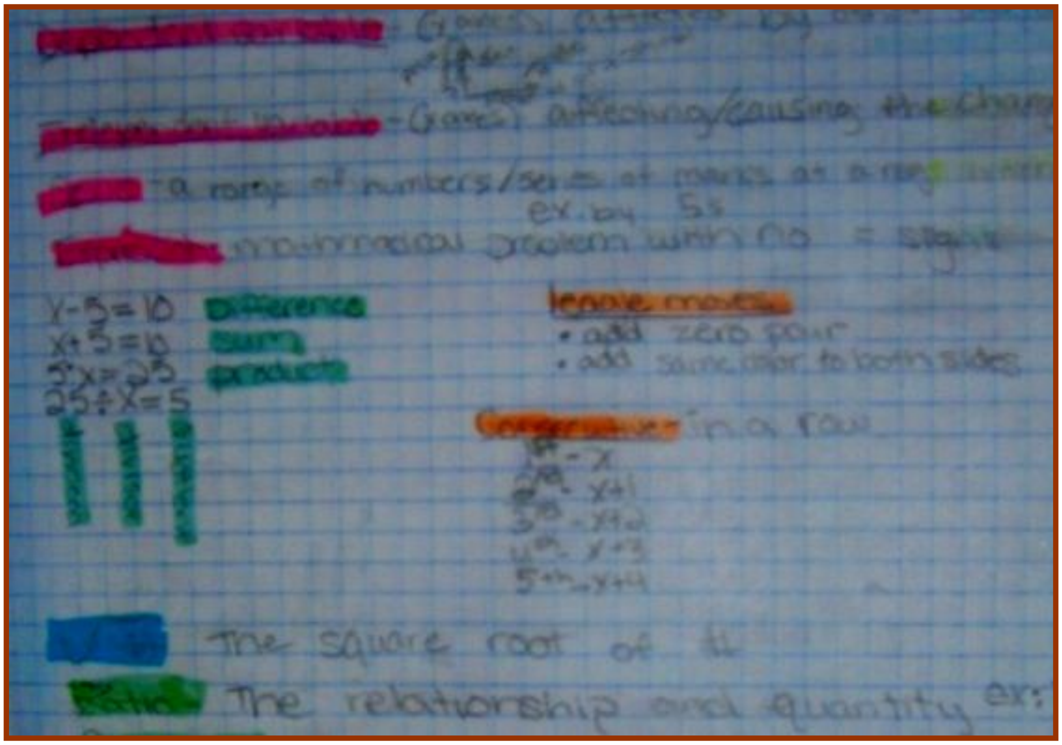


Figure 4 Personal Glossary

Personal Glossaries and Discussion Reflection

One fall day, I had a verbal problem on the warm-up board, which stated: *Christy is 3 years older than Jamie. The sum of their ages is 23. How old is each?* While students were working individually, one student asked me what *sum* was—the answer to subtraction or addition? I threw the question back at him and he said, "Subtraction…?" I lifted my eyebrows and he says, "I mean addition!" We laughed and I said "yes." He then raised his eyebrows and said, "So, is this a word I should be adding to my personal glossary?" I clapped my hands and praised him for the good idea because now he will have a reference if he needs to remind himself of the meaning of sum later in the year**.**

When I started to go over the problem with the class, another student asked what *sum* meant, and I asked the class the question. The first person to answer said it was the answer to an addition problem. Another student then countered her and said he thought it was the answer to any problem. I then put a subtraction problem (x – 5 = 10) and an addition problem (x + 5 = 10) on the board and said one was sum and one was not. They decided that x + 5 = 10 was a sum. We then got into a discussion about what the answer to a subtraction, division, and multiplication problem were called. I put examples of each up on the board. I asked, "What is the answer called to this type of problem?" While students took a few guesses at times, they usually gave a response that was one of the other answers and their classmates helped them sort it out. I also ended up discussing each part of a division problem because they threw out terms but didn't know which one was which (*divisor, dividend,* and *quotient*, words I sometimes still mix up). Most of them wrote these words in their personal glossaries, which I suggested that they do if they needed to for future reference. While the students were familiar with the words (*sum, difference, quotient,* and *product*), I was surprised that they did not know their specific definitions, so I was thrilled that it came up because of student questioning and that we were able to discuss more deeply than I had anticipated.

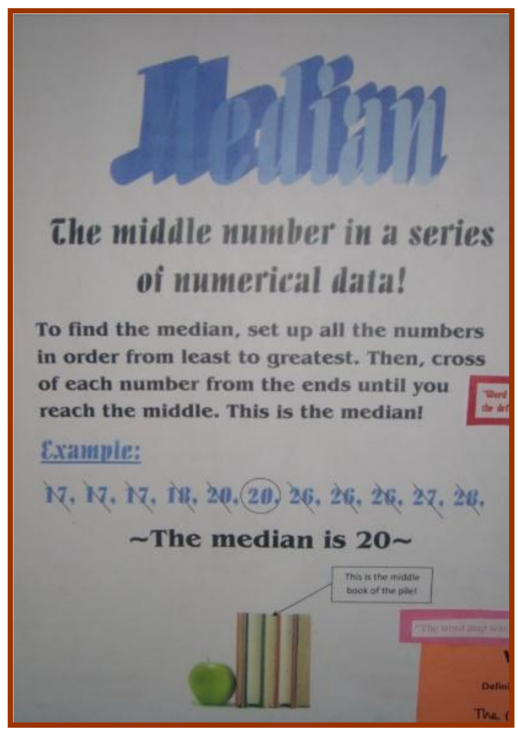


Figure 5 Poster

In my log I noted, "I am absolutely ecstatic that students are recognizing that when they do not know a word, it is hindering their ability to solve a problem and that they should add it to their glossaries for future reference" (personal log, October 9, 2008). Throughout the year, some students expressed exasperation at having to write words in their personal glossaries for various purposes and units, yet other students would flip quietly to their personal glossaries during class and write down new vocabulary on their own. One student even wrote on the final survey that the personal glossary was the math activity that best helped her understand because she "can always look back."

Word Wall

When examining the effectiveness of the word wall, I noted that 35% of students cited the activity as helpful in getting a more thorough understanding of one specific word, yet based on questions students asked me during a pretest, I wondered how much *processing of the vocabulary word wall words* was going on and *how much copying and pasting without real thinking* was going on. This lack of transfer first appeared on the *Accentuate the Negative* pretest when one student who happened to have done the word *quadrant* as his word wall word, asked me a question on the test about if it mattered which quadrant was labeled as which number and I said "yes…" Then I said, "wait, didn't you do quadrant as your word? " And he says, "yes, but I forget." I then realized that his clipart image had it labeled correctly on his poster but he must not have processed it. However, when I got his pretest, he had indeed done it correctly.

Figure 6 Word Wall

Another student who happened to have done *absolute value* as her word wall poster asked me about what absolute value meant and said she forgot when I posed the same question to her as the other student. She actually made the wrong symbol on the pretest (approximately sign) because she hadn't included the absolute value symbol on her poster but she was able to match the word correctly (I then had her add the correct symbol to her poster). Yet later in the year, another student asked me what 'mean' meant, and I told him to think back to all the measures of center but that I couldn't tell him during a test. At the end of the period, he happily stopped by my desk to tell me that it suddenly came to him because it was his word wall word. Most often, though, students showed excitement about their word wall posters and would question me about when their word would be on the wall (see Figure 1).

[I felt an anxious tapping on my shoulder in 8th period.]
Student: "Oo, Oo, Ms. Peirano, Ms. Peirano, can I create a word wall poster for linear?" [I hesitated, not sure if he was done with his class work-a test review packet for our introduction unit to algebra, which included making tables, graphs, and basic equations.]
Student: "I've finished my packet."
Ms. Peirano: "And you've checked your work with my answer key, and you have no questions?"
Student: "Nope. But can I make a poster for linear?" [His pleading continued. I excitedly agreed, but because we had not covered it in-depth, I sat down and discussed the word with him.]
Ms. Peirano: "What definition were you thinking of using?"
Student: "It means when the line is straight on the graph." [I probed him more.]
Ms. Peirano: "What type of situations have we done in class that have created linear graphs?" [Pause]
Ms. Peirano: "For example, what's happening in the Jesse/Kay problems we've been doing?"
Student: "Well they make a certain amount of money each day. And sometimes they start with money, too."
Ms. Peirano: "What is happening in the tables you've been creating in your warm-up? What do you think is special about them that makes them linear?"
Student: "They're making the same amount of money everyday. They're going up by the same number."
Ms. Peirano: "Exactly! So how can you incorporate that idea into your definition?
Student: "That it goes up by the same amount."
Ms. Peirano: "Good start, but sometimes Jesse and Kay might lose the same amount of money each day, so it won't always go up. What else could you say?"
[I excitedly helped this self-motivated student shape a definition for a word wall poster, and after he created it, he begged me for days to put it up, and finally he stayed after school one day to help put it up and then stood there and gleamed with pride.]

Figure 7 Dialogue about Word Wall (Taken from 9/26/08)

Simile/Fortune Note Cards

It took time out of our math class to fully explain this vocabulary assignment and to review similes and metaphors; however, after receiving finished products and student comments, I feel it was time well spent. Grading the simile/fortune note cards was tricky because I could not always gauge their full understanding based on the simile they had created; I created a rubric to assist in the task. Not only did I really enjoy reading students' similes and fortunes and gained tremendous insight from their written thoughts, but students enjoyed sharing what they created. When students came in the door the day the note cards were due, many asked if we would be sharing them in groups and expressed that they would like to do so. While they shared words in their groups, I was able to help with some misconceptions. The students had fun with it, too, and many were then willing to share theirs out loud with the whole class. I noted in my log, "I couldn't believe how many of them were so pumped up to share their similes or said, 'Look at this! It's a really good one!' I love to see that kind of excitement" (personal log, 10/31/09). I also noticed that students related math terms to things they like, such as football, food, or art. Using literary techniques can "bring words to life, subtly attaching relationships that deepen, clarify, and even add new meanings to quite ordinary math words" (Murray, 2004, p. 154). Analogies and metaphors are effective in helping students assimilate technical vocabulary and understand new or challenging concepts (Rubenstein, 1996; Murray, 2004).

Summary of the vocabulary activities at the end of the quarter

While the vocabulary activities stood out in my mind because of my vocabulary focus this year in mathematics, I was not sure what overall impact they had left on the students. Yet it seemed that the students, through the variety of vocabulary activities, along with the mathematics, seemed to recall specific terms, even at the end of the quarter. In my log, I wrote, *"Today in first period, my students brainstormed what we have learned this nine weeks, topics we've covered, activities we've done, etc… I never specifically told them to use or list vocabulary we've covered. They did list big ideas/topics and activities. However, the kids also told me different symbols and vocabulary words we have learned, such as delta ("a change in," which one student explained to another), approximately sign (which they described as the wavy lines), and absolute value. They also mentioned the 'key words' in word problems we had discussed, such as words that meant to multiply, add, and so on. They used the word zero pair, inverse operation, expression, consecutive numbers, whole numbers, integers, and quadrants as well as graphs, coordinate pairs, tables, and equations/rules. I was very excited that they were using all of this specific mathematics vocabulary. When I asked them the difference between expression and equation, one girl started to tell me while another one looked it up in her personal glossary. The students' enthusiasm for all we had done so far was surprising, and I was so happy with it. They made comments about how they couldn't believe how much we had learned this first quarter and a few said, "We've learned more in the first nine weeks than we learned ALL of last year!" They expressed how much fun math was and one student suggested that we make a movie of how much we've learned (and the others quickly chimed in in agreement) and how excited they were (one boy, JS, put his hands in the air like he was cheering for math and saying that it would be a great clip for the movie). One girl suggested that we use it for next year's group to get them excited about class. While we were laughing and having a good time, I reminded them that we had to use our time productively and if they really liked the idea of a 'movie,' then perhaps we could do something with our vocabulary words. JS exclaims, "Oh that will be interesting, so like for linear I do this [he puts his arms straight up in the air and makes his legs straight, tilting sideways]. Even though he was being silly, I could see from his body language that he understood linear meant that it makes a straight line. I was surprised he chose that word to attempt because we didn't cover it much in depth, but obviously something stuck with him, and correctly, at that! When they left class, I felt really good about what we have learned and how excited the kids were about their learning. And clearly, at least some of the vocabulary is sticking with them"* (personal log, 11/7/09).

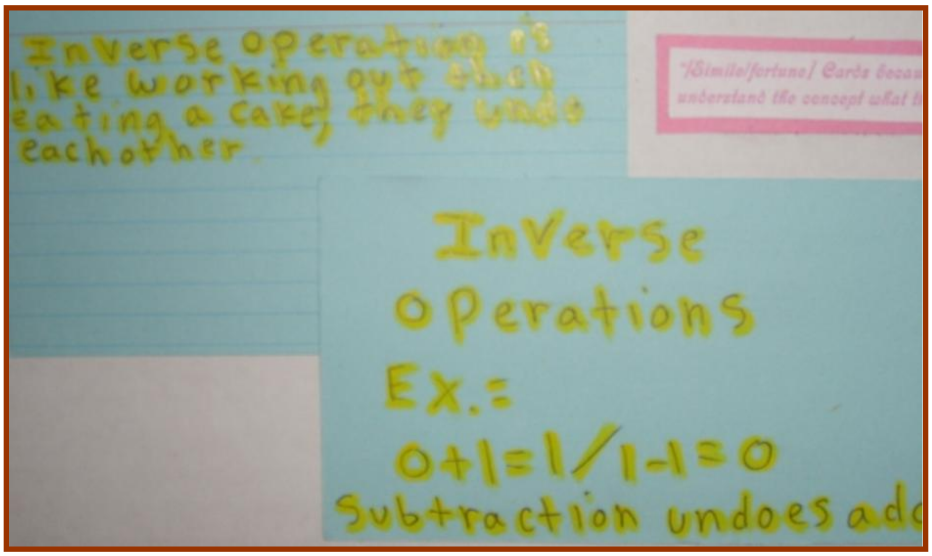


Figure 8 Simile Notecard

Discussion

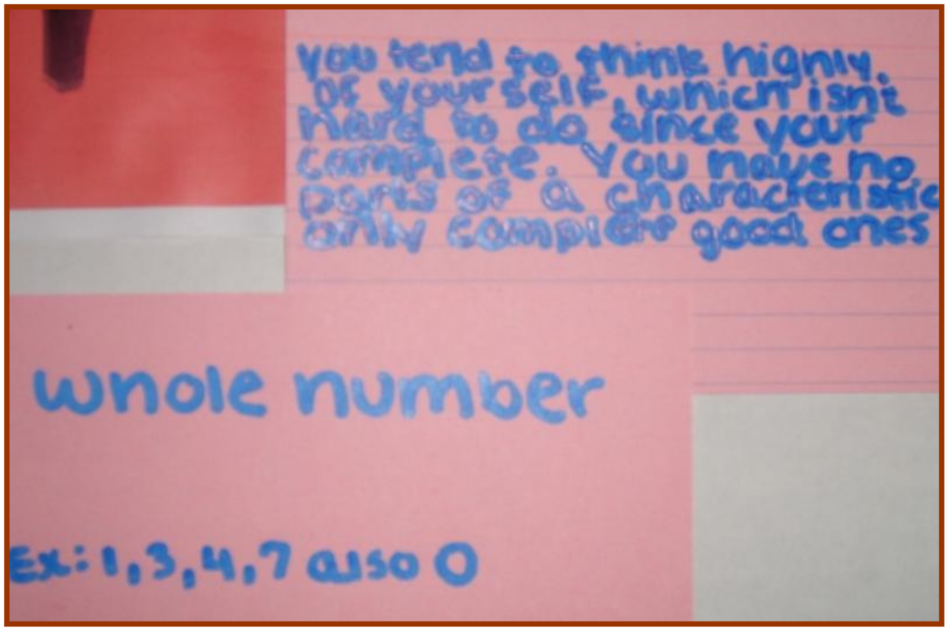


Figure 9 Fortune Note Card

The Role of Symbolic Vocabulary in Understanding Mathematics

While my focus in this study was on verbal vocabulary, I've to come to realize that *symbolic* vocabulary plays just as critical a role in helping students comprehend mathematics and properly solve problems. I noticed that while students might be able to define absolute value correctly and list two numbers that have a certain absolute value, students still struggled with the absolute value *symbol*. If students do not understand what the symbol is telling them to do, much like if they do not understand the meaning of a mathematical term, then it seems they are just as likely to get a problem incorrect because of their lack of understanding of mathematical symbols. Because mathematics comprehension includes not only understanding the words but also the symbols and concepts, it becomes important that students understand mathematical symbols, their meanings, and how to utilize them to comprehend and solve problems. "Mathematics does not lie in its symbols, but in the ideas these symbols represent" (Davis, 1986, as cited in Capraro, 2006, p. 162). One of my students even brought up the importance of viewing symbols as vocabulary when he wrote on a survey that the vocabulary method that helps him the most is "when I actually use it [vocabulary.] Like absolute value, we had to learn the sign and then know what it meant." Further research could be done on this topic because of the common use of symbols as a way to convey mathematical meaning.

Struggles in the learning process with vocabulary definitions and examples

Based on student interviews, surveys, and test results, it appears to be important to teach vocabulary concepts and definitions simultaneously. Student comments about their struggle to find definitions and examples for the different vocabulary strategies included, "I had trouble finding the definition because they didn't make sense," "They weren't math terms," "I couldn't find the definition with an example," "I'm not always able to make up examples [just based on the definition]" and that "it helps me [when you give us the definitions] because I don't always understand what they [dictionaries] are trying to say. They use big vocabulary." Another student said she was more comfortable when I provided definitions because it's a definition she knows is right. I even noticed when grading students' graphic organizers that sometimes students were too vague or struggled to write a definition in their own words. Students sometimes used the words they were defining in their definitions.

Learning vocabulary is not just about determining one straightforward definition but is also about examining the meanings of words through broader concepts. "Definitions alone rarely throw much light on the ideas they represent. They are usually the end product of much exploration and careful thought. In fact, precision of a definition belies the effort that has contributed to its formulation" (Countryman, 1992, p. 55, as cited in Murray, 2004). Because of the complexity of mathematical terms and the concepts they represent, teachers will want to build concepts first and then attach mathematical vocabulary to established ideas (Lee, 2007), which is also why it is important that students do not use dictionaries until after concepts are represented and understood (Blachowicz and Fisher, 1996, as cited in Murray, 2004). Only after other efforts to acquire mathematical meanings have been exhausted should students utilize a math dictionary (Garbe,1985, as cited in Adams, 2003).

Examining Mathematical vs. Everyday Meanings of Vocabulary

After reading over the vocabulary section of students' first pretest, I had noted that I wanted to discuss that some words have 'math' meanings and 'everyday' meanings or even different meanings in different areas in math. For example, with the word "scale," they used their prior knowledge of scales in hands-on equations from last year to talk about a balance as opposed to the range of numbers on the axes of a graph (personal log, 9/8/08). One student even drew a scale that people use to weigh themselves on and another drew a kitchen scale for measurement. One student even told me in an interview that when she tried to look up definitions in the dictionary, "They weren't math terms. I was tempted to write the general definition"; at least she recognized the difference. However, research states that one reason that learning mathematics vocabulary is difficult is that students must learn new meanings of math vocabulary that have completely different meanings from what they already know (Capraro, 2006, Carter and Dean, 2006, Graves as cited in Harmon, 2005), which can be confusing for students (Rubenstein, 2007). Teachers can be aware of potential confusion and help students see the mathematical meaning and compare it to the everyday meaning, examining both similarities and differences. Through discussion students can learn why a word in a particular context may have a specific meaning (Rubenstein, 2007; Lee, 2007). When dealing with multiple meaning words, the teacher can make connections between students' prior understandings of the word and the mathematical meaning of the word so that students can begin to develop definitions from their own experiences (Adams, 2003). However, their prior knowledge may sometimes actually get in the way because the words may be completely unrelated (Kossack, 2007).

Another observation I made was that if students do not know regular, everyday vocabulary, they might also struggle to correctly solve math problems. In my log, I recorded an incident when several students expressed frustration with not knowing whether to add or subtract on a series of business transactions from a problem in their textbook, which included words like *refund, refund from return, down payment, sale, business insurance, advertising*, and so on (personal log, 10/24/08). In another entry, on one day two students asked about the difference between *consecutive, consecutive even, consecutive odd*. Two other students asked the meaning of *consecutive*. One student asked the meaning of *diminished* and another asked about what 'obtain' meant, all words that would not be considered math specific (personal log, 10/22/08).

Conclusion

"I realized that I am now more conscious about using mathematical vocabulary when having whole class, small group, and one-on-one discussions. I think that I am realizing the more they are exposed to the correct terminology in context, the more likely they are to process it and use it correctly" (personal log, 9/27/09). Towards the end of the school year, I began a unit by calling the number in front of the x-variable just that. A week later, I began to then call the number in front of the x-variable its proper name—"coefficient." As I introduced the term to the students, one student asked, "Well why didn't you just call it that from the beginning?" This served as a reminder that the kids want and need to know mathematical vocabulary and that the more they're exposed to it, the more it is to be embedded with the concepts.

A deeper awareness of the importance that vocabulary plays in the math classroom is necessary in order for teachers to help students make connections between the various math concepts and the terminology associated with those concepts. This study reiterates that teachers of mathematics are just as much teachers of literacy. There are many strategies described in the literature for teaching vocabulary in math, so for a more comprehensive list of ideas, see Table 3.

Table 3 MATHEMATICAL VOCABULARY STRATEGIES

Examining mathematical vs. everyday meanings of vocabulary	*Avoiding homonym confusion*	*Approaching mathematic vocabulary instruction using linguistics*	*Moving from informal to formal definitions*	*Utilizing multiple intelligences to strengthen vocabulary*	*Sorting out vocabulary in different areas of mathematics*
Compare it to the everyday English word, examining both commonalities and differences (Rubenstein, 2007)	**Acknowledge that other words sound the same, identify those other words, say and spell each, define them, and use each in context (Rubenstein, 2007)**	**Examine words' historical etymologies and relationships (Rubenstein, 2007)**	**Explore the general case (an unmodified term, e.g. trapezoid) and then the specific case (a modified term, e.g. isosceles trapezoid) (Rubenstein, 2007)**	**Use picture dictionaries as a visual learning strategy in which pictures are connected with written descriptions in the students' own words (Lee, 2007)**	**Establish a list of math vocabulary for each area of mathematics (Adams, 2003, Lee, 2007)**
Make connections between students' prior understandings of the word and the mathematical meaning of the word so that students can begin to develop definitions from their own experiences (Adams, 2003)	**Keep a homonymic bulletin board or a list of homophones (Adams, 2003)**	**Analyze the meanings of words through prefixes, suffixes, and roots (Lee, 2007)**	**Start with informal definitions and construct one's own meaning (Leung, 2005)**	**Create math-oriented cartoons or perform skits that involve mathematical ideas (a visual , kinesthetic, and artistic opportunity for students who struggle with written or verbal strategies) (Lee, 2007)**	**Color code terms to correspond to the various areas of math using the same terminology that appears on the standardized tests (Lee, 2007)**
Use discussion to discover the rationale behind why a particular word may have its meaning (Lee, 2007)	**Assign a journal prompt that requires students to examine specific words that have homophonic partners (Adams, 2003)**		**Look at examples and non-examples to establish formal definitions (Adams, 2003)**	**Associate meaning with 'memorable vivid images' (Kossack, 2007)**	

Table 3 MATHEMATICAL VOCABULARY STRATEGIES, continued

Writing in math to deepen vocabulary and conceptual understanding	*Creating graphic organizers*	*Interacting to learn vocabulary*	*Assessing vocabulary in math*	*Learning Math Symbols*	*Using literature in math*
Write journals about the day's lesson, including in-class problems, notes, and daily reflections (Murray, 2004)	**Use concept maps to organize info and vocabulary visually as well as make connections between concepts (Murray, 2004, Harmon, 2005)**	**Have students highlight challenging vocabulary and discuss their understandings in small groups. Then, the teacher can address any words that still need clarification (Leung, 2005)**	**Utilize mathematical reflections in which students use vocabulary to discuss understanding of concepts (Murray, 2004)**	**Direct students' attention to numerals in context and have them classify numbers in a variety of contexts, justifying their decisions (Capraro, 2006)**	**Give students opportunities to practice vocabulary and see it in context outside of the math textbook, such as through picture books (Harmon, 2005)**
Write poetry, encouraging the use of analogies & metaphors to help assimilate technical vocabulary and understand new or challenging concepts (Rubenstein, 1996, Murray, 2004)	**Brainstorm lists of words related to the topic of the lesson, using graphic organizers to diagram relationships between words (Harmon, 2005)**	**Try to include key vocabulary terms/concepts when questioning students & ask for explanations, justifications, & conjectures so that students, in turn, must use the vocabulary and expand on their thinking (Leung, 2005)**	**Use self-assessment to examine how students have grown as mathematical learners (Adams, 2003)**	**Play matching games, such as drawing a symbol from the deck of cards in which a correct math sentence, including the symbol, must be stated in order to win the card (Lee, 2007)**	**Have trade books in the classroom that are topically related to concepts being studied so that struggling readers, in particular, can access the info (Harmon, 2005)**
Keep a math vocabulary section in their binders in which they define the math term in their own words and use an illustration and example (Murray, 2004, Harmon, 2005)	**Map words, sorting words into categories, categorizing words into commonalities (Murray, 2004)**	**Use the whole-class activity "I Have, Who Has?", a vocabulary game in which one student reads a definition and the student with the corresponding definition responds and then reads his/her word, until all terms have been exhausted and the class is back to the student who started (Rubenstein, 2007)**		**Play a game in which each person begins with 5 symbol and numeral cards in their hand and they much make correct mathematical expressions until all cards are gone, even those in the center pile (Lee, 2007)**	

Table 3 MATHEMATICAL VOCABULARY STRATEGIES, continued

Writing in math to deepen vocabulary and conceptual understanding	*Creating graphic organizers*	*Interacting to learn vocabulary*	*Assessing vocabulary in math*	*Learning Math Symbols*	*Using literature in math*
Keep a personal word wall, which lists alphabetically all the terms used throughout the school year (Murray, 2004)		**Acknowledge that some math terms are easily confused, such as factor and multiple, and engage in class discussion about ways they can be distinguished and remembered better (Rubenstein, 2007)**			

The better students understand mathematical vocabulary, the more likely they are to succeed in mathematical applications and problem-solving. As stated by the students themselves, "You should [know math vocabulary] so you can explain situations using math terms and you have to understand them for work and school" and "if you don't know vocabulary from math you won't fully understand what you're learning."

For some it may seem that an emphasis on vocabulary in the math classroom will detract from important content learning; however if there is a balance then the work with vocabulary will pull in all other concerns and appears to do so effectively (Murray 2004). Mathematics, in its complexity, cannot be fully understood without in-depth exploration of the specialized vocabulary and the concepts associated with the vocabulary.

> The words, symbols, and numerals that give the discipline its substance, framework, and power are the same words, symbols, and numerals that students must use to communicate ideas, perform procedures, explain processes, and solve problems. Hence a knower of mathematics is a doer of mathematics, and a doer of mathematics is a reader of mathematics (Adams, 2003, p. 792)

Teachers need to play an active role in helping students understand how to problem solve, both as a reader and a reader of mathematics. "Instruction that helps learners view mathematics as a tool for solving problems, participating in recreation and other pleasurable activities, and making sense of the world as the learner sees it is instruction that motivates students to read mathematics" (Adams, 2003, p. 792). And only when students *read* mathematics can they investigate the meaning of words, and only when they investigate the meaning of words can they truly begin to learn about, explore, and discover the complexities of our mathematical world.

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